# Rotations

In spite of copious tutorials on rotations available on the Internet describing how to manipulate quaternions, Euler angles, etc, the author has been frustrated for years by the scant discussion of what a rotation really *is.* Often times the definitions one finds are vague or circular, speaking in terms of *revolving* around some *axis*, without necessarily defining an axis. Other times definitions involve trigonometry or other proxies for what the author would call *parameterization*, not definition.

This work seeks to state a formal definition of rotation in 3D Euclidean space in terms only of vector algebra, thus achieving the end of precise discussion while relying for foundation only on concepts familiar to anyone who has studied elementary linear algebra.

We start by reviewing some Preliminaries of vectors, matrices, and trigonometry. Then we give our definition followed by a listing and by proofs of many of the properties of rotation, showing that these properties follow from our definition. Then we establish several of the familiar parameterizations and show that these all satisfy out definition. Finally we go over much of the computation that is possible among the parameterizations we have established.